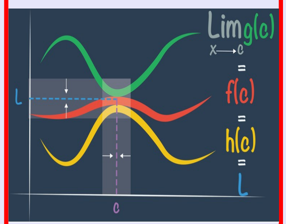
Math 261

Spring 2022

Lecture 15



Given $y = \sqrt{2x+1}$, where x & y are functions of t

Find $\frac{dy}{dt}$ if $\frac{dx}{dt} = 3$ and $x = 4$. $\to y = \sqrt{2(4)+1} = 3$

$$y^2 = 2x + 1$$

$$2y\frac{dy}{dt} = 2\frac{dx}{dt} + 0$$

$$2(3)\frac{dy}{dt} = 2 \cdot 3 \Rightarrow \boxed{\frac{dy}{dt} = 1}$$

Find $\frac{dx}{dt}$ if $\frac{dy}{dt} = 5$ and $x = 12$. $\to y = \sqrt{2(12)+1} = \sqrt{25} = 5$

$$2y\frac{dy}{dt} = 2\frac{dx}{dt}$$

$$y\frac{dy}{dt} = \frac{dx}{dt}$$

$$5 \cdot 5 = \frac{dx}{dt} \quad \boxed{\frac{dx}{dt} = 25}$$

March 30, 2022

Given $x^2 + y^2 + z^2 = 9$

x, y, and z are functions of t.

$\frac{dx}{dt} = 5$, $\frac{dy}{dt} = 4$, find $\frac{dz}{dt}$ at the

Point $(2, -2, 1)$.

1) Verify the point. $(2)^2 + (-2)^2 + 1^2 = 9$

$4 + 4 + 1 = 9$ ✓

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} + 2z\frac{dz}{dt} = 0$$

Divide by 2

$$x\frac{dx}{dt} + y\frac{dy}{dt} + z\frac{dz}{dt} = 0$$

$$2 \cdot 5 + (-2) \cdot 4 + 1 \cdot \frac{dz}{dt} = 0$$

$$10 - 8 + \frac{dz}{dt} = 0 \Rightarrow \boxed{\frac{dz}{dt} = -2}$$

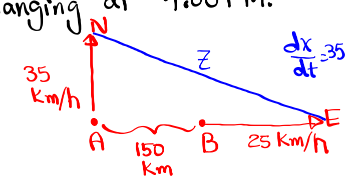
At noon, ship A is 150 Km west of ship B.

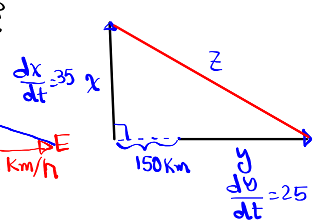
Ship A is sailing north at 35 Km/hr.

Ship B is sailing east at 25 Km/hr.

How fast is the distance between them

changing at 4:00 PM?





Using Pythagorean thrm

$$x^2 + (150 + y)^2 = z^2$$

$$\cancel{2}x\frac{dx}{dt} + \cancel{2}(150 + y)\cdot\frac{dy}{dt} = \cancel{2}z\frac{dz}{dt}$$

At 4:00 PM

$x = 4(35) = 140$

$y = 4(25) = 100$

$140^2 + (150 + 100)^2 = z^2$

$82100 = z^2$

$z = \sqrt{82100}$

$z = 286.5$

$$140 \cdot 35 + (150 + 100) \cdot 25 = 286.5\frac{dz}{dt}$$

$\boxed{\frac{dz}{dt} = \quad \text{Km/h}}$

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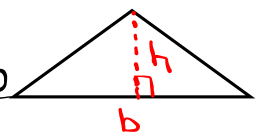
The height of a triangle is increasing at the rate of 1 cm/min $\frac{dh}{dt}=1$ while the area is increasing at a rate of 2 cm²/min. $\frac{dA}{dt}=2$

At what rate its base changing when the height is 10 cm and the area is 100 cm²?

$A=\frac{bh}{2}$



$100=\frac{b\cdot 10}{2}$

$b=20$

$A=\frac{bh}{2}$ $\quad 2A=bh$

$$2\frac{dA}{dt}=\frac{db}{dt}\cdot h+b\cdot\frac{dh}{dt}$$

$$2\cdot 2=\frac{db}{dt}\cdot 10+20\cdot 1$$

$$\boxed{\frac{db}{dt}=-}$$

base is decreasing

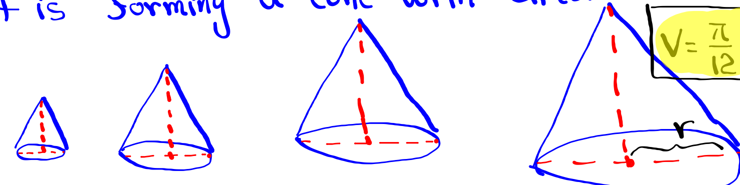
Gravel is being dumped at the rate of 30 ft³/min. $\frac{dV}{dt}=30\ ft^3/min.$

It is forming a cone with circular base

$V=\frac{1}{3}\pi r^2 h$

$V=\frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$\boxed{V=\frac{\pi}{12}h^3}$



height of the cone is always equal to the diameter of its base. $h=d$ $\quad h=2r \rightarrow r=\frac{h}{2}$

How fast is the height increasing when it is 10 ft high?

$$V=\frac{\pi}{12}h^3$$

$$\frac{dV}{dt}=\frac{\pi}{\cancel{12}}\cdot \cancel{3}h^2\cdot\frac{dh}{dt}$$

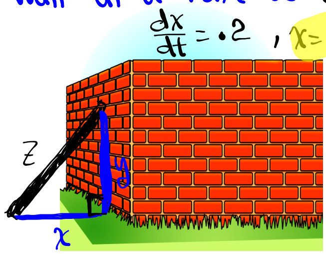
$$30=\frac{\pi}{4}(10)^2\cdot\frac{dh}{dt}$$

$$\frac{dh}{dt}=\frac{6}{5\pi}$$ ft/min

A ladder is leaning against a vertical wall. The top of the ladder slides down at a rate of .15 m/s. $\boxed{\frac{dy}{dt}=-.15}$

At the moment when the bottom of ladder is 3 m from the wall, it slides away from the wall at a rate of .2 m/s. How long is the ladder?

$\frac{dx}{dt}=.2$, $x=3$



$x^2+y^2=z^2$ — Constant

$\cancel{2}x\frac{dx}{dt}+\cancel{2}y\frac{dy}{dt}=0$

$3\cdot(.2)+y\cdot(-.15)=0$

$-.15y=-.6$

$y=\frac{-.6}{.15}$ $\boxed{y=4}$

Find z $x^2+y^2=z^2$

$3^2+4^2=z^2$

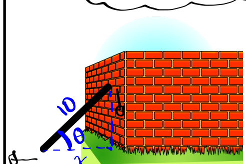
$\boxed{z=5m}$ 5 m long

A ladder 10 ft long is resting against a vertical wall.

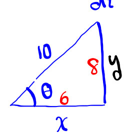
The bottom of the ladder slides away from the wall at a rate of 1 ft/s.

How fast is the angle between the ladder and the ground changing when the bottom of the ladder is 6 ft from the wall?

find $\frac{d\theta}{dt}$ when $x=6$



$\frac{d\theta}{dt}$ $\frac{dx}{dt}=1$ ft/s.



$x^2+y^2=10^2$ when $x=6$ $y=8$

$\sin\theta=\frac{y}{10}$

$\boxed{\cos\theta=\frac{x}{10}}$ $10\cos\theta=x$

$\tan\theta=\frac{y}{x}$

$10\cdot(-\sin\theta)\cdot\frac{d\theta}{dt}=\frac{dx}{dt}$

$-\cancel{10}\cdot\frac{8}{\cancel{10}}\cdot\frac{d\theta}{dt}=1$

$\frac{d\theta}{dt}=-\frac{1}{8}$ Rad/s.

Angle θ is decreasing

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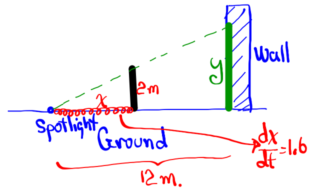
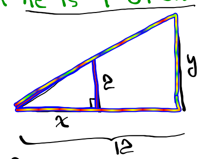
A spotlight on the ground shines on a wall 12 m away.



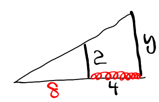
A man is 2m tall walks from the spotlight towards the wall at a speed of 1.6 m/s

How fast is the length of his shadow on the wall changing when he is 4 ft from the wall?

$\frac{dy}{dt} = -$



$\frac{x}{2} = \frac{12}{y}$ $\quad xy = 24$



$\frac{8}{2} = \frac{12}{y}$ $\quad \frac{dx}{dt} y + x \frac{dy}{dt} = 0$

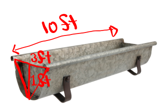
$y = 3$ $\quad 1.6 \cdot 3 + 8 \cdot \frac{dy}{dt} = 0$

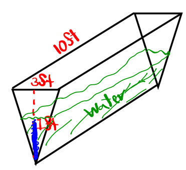
$4.8 + 8 \frac{dy}{dt} = 0$ $\quad \frac{dy}{dt} = \frac{-4.8}{8}$

$\frac{dy}{dt} = -.6$

length of shadow is decreasing at .6 m/sec.

A trough is $\boxed{\text{10 ft long}}$ and its ends have the shape of isosceles triangles that are 3 ft across at the top and have a height of 1 ft.





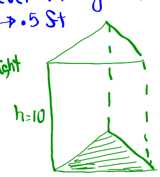
Not Scaled

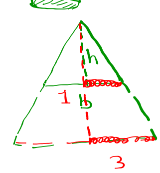
Water is being poured in at a rate of 12 ft³/min. $\quad \frac{dV}{dt} = 12$,

Volume ?
Volume = Area of base • hight

How fast is the water level rising when the water is 6 in deep? → .5 ft

V = Area of base • height
$= \pi r^2 \cdot h$





V = Area of base • height
$= \frac{b \cdot h}{2} \cdot 10$
$= 5 b h$

$\frac{h}{\frac{b}{2}} = \frac{1}{1.5}$

$V = 5 \cdot 3h \cdot h$

$1.5h = \frac{b}{2}$ $\quad \frac{3}{2}h = \frac{b}{2}$ $\quad b = 3h$ $\quad V = 15h^2$

$V = 15h^2$

$\frac{dV}{dt} = 15 \cdot 2h \frac{dh}{dt}$

$12 = 30 \cdot (.5) \frac{dh}{dt}$

6 inches = .5 ft

$12 = 15 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{12}{15} = \frac{4}{5} = .8$

$\frac{dh}{dt} = .8$ ft/min.

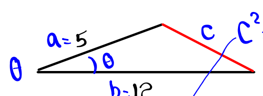
Two sides of a triangle are 12m & 5m.

The angle between them is increasing at 2°/min.

How fast the third side changing when the angle is 60°?

Law of Cosines

$c^2 = a^2 + b^2 - 2ab\cos\theta$



$c^2 = 25 + 144 - 120\cos\theta$

$\theta = 60^\circ$ $\quad \cos 60^\circ = \frac{1}{2}$

$c^2 = 25 + 144 - 120 \cdot \frac{1}{2}$

$= 169 - 60 = 109$

$c = \sqrt{109}$

$c^2 = 169 - 120\cos\theta$

$\cancel{2}c \frac{dc}{dt} = -\cancel{120} \cdot (-\sin\theta) \cdot \frac{d\theta}{dt}$

$c \frac{dc}{dt} = 60 \sin\theta \frac{d\theta}{dt}$

$\frac{d\theta}{dt} \to 2^\circ/\text{min}$

$2 \cdot \frac{\pi}{180}$ Rad/min.

$\sqrt{109} \frac{dc}{dt} = 60 \cdot \sin 60^\circ \cdot \frac{\pi}{90}$

$\boxed{\frac{dc}{dt} = \quad \text{m/min.}}$

Class QZ 10

Use linear approximate to estimate $\sqrt[3]{1.1}$. Round to 3-decimals.

$f(x) = \sqrt[3]{x}$ $\quad a = 1$ $\quad f(1) = 1$

$f'(x) = \dfrac{1}{3\sqrt[3]{x^2}}$ $\quad f'(1) = \dfrac{1}{3}$

$L(x) = f(1) + f'(1)(x-1)$

$L(x) = 1 + \frac{1}{3}(x-1)$

$L(1.1) = 1 + \frac{1}{3}(1.1-1) = 1 + \frac{1}{30}$

$= \boxed{\frac{31}{30}} \approx \boxed{1.033}$